APMA 1200 – Operations Research: Probabilistic Models

Spring 2023

Course Syllabus and General Information

Instructor
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Course Schedule: Tuesday and Thursday, 1:00–2:20 pm, Room 130, 85 Waterman.

Course Website: https://canvas.brown.edu/courses/1090748

Course Description. This course serves as an introduction to stochastic processes and stochastic optimization. After a review of basic probability theory, including conditional probability and conditional expectation, topics covered will include discrete-time Markov chains, exponential distributions, Poisson processes and continuous-time Markov chains, elementary queueing theory, martingales, Markov decision processes and dynamic programming, with a discussion of applications.

Prerequisites. APMA 1650 (or equivalent) and MATH 520 (or equivalent). The course requires calculus, basic probability theory, and linear algebra.

Course Syllabus. The plan is to do 1-5, and and perhaps a topic from 6 depending on the interests of the class and time available.

1. Review of probability
    (a) The mathematical setup: sample space, events and probabilities
    (b) Random variables, expected value and induced distributions
    (c) Joint distributions and conditioning
    (d) Variance, covariance and independence
    (e) Some limit theorems
2. Discrete time Markov chains
    (a) Examples of Markov chains, and a non-Markovian example
    (b) General setup for finite state Markov chains
    (c) Explicit construction of a chain with given characteristics
    (d) Chapman-Kolmogorov equations
    (e) Computing expected values via first step analysis
    (f) Stationary distributions
    (g) Strong Markov property and classification of states
    (h) Reversible chains
    (i) Countable state chains
3. Continuous time Markov chains
    (a) The role of the exponential distribution
    (b) Poisson processes
    (c) Definition of a continuous time Markov chain and examples
    (d) Explicit construction of a chain with given characteristics
    (e) Relations between continuous and discrete time chains
    (f) Examples from queueing theory
4. Martingales
    (a) Definitions and examples
    (b) The Optional Sampling theorem
    (c) Applications: gambling, exit problems

5. Markov decision processes and dynamic programming

   (a) Standard costs and their characterization for uncontrolled chains
   (b) Optimal stopping
   (c) The Bellman equation for stochastic optimal control

6. Filtering of hidden Markov chains; Brownian motion

Course Objectives. The course has two objectives:

(a) to teach the basic theory and techniques of elementary stochastic processes and associated probability models;

(b) to introduce several applications of the theory and techniques. The emphasis will be on modeling, probabilistic thinking, and problem solving. The goal is to equip you with enough background knowledge on the subject so that you may explore more of the theory and applications of stochastic operations research.

Course Notes. Lectures will use both blackboard and video projection. In particular, a few more detailed examples and arguments will be done at a blackboard. Notes that will be presented in video format will be made available on the canvas page the day before, with appropriate spaces for the material to be presented at the board.

Ed Discussion page. There will be an Ed Discussion page linked to the Canvas page, and used for various questions, clarification of any confusing statements in homework, etc.

Reference Books. For those looking for details that may have been omitted in class, more examples, alternative derivations, etc., three books to consider are the following:

- S. Ross, Introduction to Probability Models, 10th Edition, Academic Press, 2010.
- R. Durrett, Essentials of Stochastic Processes, 3rd Edition, Springer, 2016.
- J.R. Norris, Markov Chains (electronic version via Canvas link)

The first book is the most elementary and covers everything we will discuss except dynamic programming and martingales. The second is more advanced and has a chapter on martingales. Electronic versions of the first two books can be downloaded from the library. The last book is the most advanced, and is popular with students who go further into the material than is possible in an introductory course. Links to all three appear on the Canvas page, though as mentioned the first two can also be downloaded.

Homework. Homework will be assigned on a roughly weekly basis. Unless otherwise noted, it will be posted on Canvas by Friday and due the next Friday at 5 PM. Homework should be submitted through Gradescope. Unless explicitly stated otherwise, students can discuss concepts behind the homework with each other. However, each student must independently write down their solutions and any explicit help taken from fellow students or other source should be explicitly acknowledged. As a general rule students are expected to follow the Brown Academic Code. The lowest homework grade will be dropped.

Late assignments are not accepted, unless explicit permission is given for a valid reason at least one day prior to the due date of the homework.

Quizzes. To help keep everyone on the same page, quizzes requiring only a true/false response will be given approximately once a week. These will require little calculation, and are intended to check familiarity with recently covered definitions, theorem statements, etc. Since some absences from class will occur, the best 80% of quiz grades will be used for the overall quiz grade. If unusual circumstances arise that may require further consideration then a dean's note will be needed.

Exams and Grading. The weights for the final class grade are:

- Homework (average of all homeworks save one) – 20%
- Quizzes – 20%
- Midterm Exam (23 March, in class) – 30%
- Final Exam (13 May, 9-12 AM) – 30%

Attendance and Course Etiquette.

- Regular attendance and completion of homework is expected of all students, and is the key to positive learning outcomes in this course. It is hoped that at this time we can return to pre-covid expectations, but of course if a major problem arises, either for individual students or for the larger community, we will make adjustments.
- Cell phones should be turned off during class; computers/I-Pads, etc. should be used only for the purpose of taking class notes.
- Students are encouraged to ask questions, during lecture, office hours and recitations.
- All e-mails to TAs/Instructor should include "APMA 1200" in the subject line

Accommodations. Students needing special accommodation should contact the instructor at the earliest, preferably in the first two weeks of the semester.

Regular Feedback Encouraged: Students are encouraged to provide timely and constructive feedback regarding the course on a regular basis: e.g., commenting on pace, level of difficulty, audibility, visibility and clarity of handwritten lectures, particular concepts that seem unclear, etc.