

MATH 1260: Complex Analysis

Professor: Nathan Wagner

Fall 2023

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Office Hours: M W 11am-12pm

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TA Office Hours: Tuesday 4-5:20pm

Room TBA

Website: <https://sites.google.com/view/nathanawagner/home>

Class Time: MWF 2-2:50pm

Class Room: Barus & Holley 158

Canvas URL: <https://canvas.brown.edu/courses/1093173>

Course Description

Complex analysis is one of the cornerstones of mathematics. Its modern form traces back to the 1800s and the work of Augustin-Louis Cauchy and several other mathematicians. Put simply, complex analysis is the study of differentiable functions of a complex variable $z = x + iy$. An example of such a function is $f(z) = z^2 + i$; indeed, any polynomial in z is an example. Complex analysis treats familiar topics in calculus, such as differentiation, integration, limits, and infinite series, in the complex variable setting and with the rigorous formulations and definitions of analysis. An overarching theme of complex analysis is that being complex differentiable (holomorphic) is a very strong condition that leads to an orderly and beautiful theory of complex functions that is also very geometric. This theory has found a myriad of applications outside mathematics, including physics and electrical engineering, and also connects to other areas of pure mathematics such as algebraic geometry and number theory.

In this course, we will cover the basics of single variable complex analysis. We will start with the basics of complex numbers, functions, and derivatives, as well as the computation of contour integrals. As the semester goes on, we will progress to foundational results such as the Cauchy integral formula, the maximum modulus principle, the open mapping theorem, Cauchy's theorem, and the residue theorem.

This is advanced undergraduate course that will entail a significant time commitment. You are expected to spend approximately 12 hours per week on this course (including 3 lecture hours, so about 9 hours out of class). You very well may find that you need to spend more than 9 hours outside of class per week on the homework and studying the material to really grasp the concepts. Learning math is not easy but it can be incredibly rewarding, and I am always here to help!

Required Materials

- Textbook: Complex Analysis (Undergraduate Texts in Mathematics) by Theodore Gamelin. Note that a free electronic copy is available through the Brown University Library at <https://library.brown.edu>.

Additional Reference Texts

The following textbooks are not required, but may be helpful during the course:

- Complex Analysis by Bak and Newman
- A First Course in Complex Analysis by Beck, Marchesi, Pixton, and Sabalka

Prerequisites

The official prerequisite for this course is multivariable calculus (either MATH 0180, 200, or 350). However, you should have some experience with writing mathematical proofs. If you do not understand what constitutes a rigorous proof, you likely need additional preparation (such as MATH 0540 or 0750) before taking this course. Please feel free to contact me or come to my office to discuss any issues related to course preparation.

Learning Goals

By the end of the course, you should be able to

1. Understand the statements of major theorems in complex analysis;
2. Apply complex analysis theorems to prove results, perform computations, and generally solve problems;
3. Critically evaluate arguments and write correct mathematical proofs with full rigor;
4. Have a general appreciation for complex analysis as an important branch of mathematical science.

Course Structure

Course Content Outline

We will cover most of Part I and Part II from Gamelin in class, and possibly portions of Part III. A rough (maybe overly ambitious) plan for the semester is as follows:

- Chapters I-V before midterm.
- Chapters VI-XI and selected topics from XIII, XIV, XV after midterm

A less ambitious plan for the semester is as follows:

- Chapters I, II, III, and IV before midterm
- Chapters V, VI, VII, VIII, and portions of IX, X, XI after midterm

Important dates:

- Every Wednesday starting 9/13 at 11:59pm, homework will be due.
- October 9: Indigenous Peoples' Day
- October 18: Midterm exam
- No class November 22 and 24 (Thanksgiving recess)
- December 8: Beginning of Reading Period, classes end (I may hold a review session during the Reading Period)
- December 16: Final exam

Here is a link to Brown's Academic calendar:

<https://www.brown.edu/about/administration/registrar/academic-calendar>

Class Structure

The primary mode of instruction in this course will be interactive lectures with ample class participation. Lecture notes will be posted to Canvas, so you won't need to take notes during class unless you prefer to do so (I found as a student that I could process the material presented in class better in real time by simply paying attention and not furiously copying notes down). You are strongly encouraged to attend these. I will also from time to time create worksheets or think-pair-share type exercises (these will not be graded, but will be factored into your participation grade if you work on them during class).

Homework

There will be weekly homework assignments posted on Canvas. These assignments will be due on Wednesdays at 11:59 pm. You will submit your solutions electronically on Gradescope. You are strongly encouraged to write your solutions in LaTeX, but it is not required. Homework will be graded for both completion and accuracy.

Exams

There will be one midterm exam (worth 20% of your grade) held in class on October 18 and a final exam (worth 30% of your grade) held during the university exam period. These exams will be written, not multiple choice, and will involve both computational and proof-based questions. The final will be cumulative and will be held at the university scheduled time (December 16 at 2pm).

Final Project

There will be a final project assignment towards the end of the semester. It will involve researching a theorem/result in complex analysis and writing a short paper (1-2 pages). The write-up should contain some background historical information, including the mathematician(s) who first proved the theorem, a statement of the theorem, and a (rough) outline of its proof in your own words. The aim is to learn some more mathematics and some interesting historical facts along the way—hopefully it will be fun! More info will be shared later in the semester.

Grading Policy

The percentages I will use in calculating grades are shown below. If necessary, I will curve grades at the end of the semester to achieve a reasonable grade distribution (which for me means the median grade lies at a minimum somewhere in the "B" range). Curves can only help you: a grade of 90% and above will guarantee you an A, 80% and above will guarantee you a B, etc.

- 35% Homework
- 20% Midterm Exam
- 30% Final Exam
- 10% Project
- 5% Participation

For students taking the course S/NC, a grade of 70% or higher will guarantee an S, while a 90% or higher will guarantee you and S_DIST (S with distinction).

Course Policies

Attendance Policy

Attendance will be worth 5% of your final grade. Positive actions that will improve your attendance grade include attending lectures, asking questions in lecture, making an effort on any worksheets I hand out in lecture, answering questions in lecture, collaborating with others on homework assignments, and attending office hours.

Late Assignments

Late work will, in general, not be accepted for full credit. Specific exceptions may be made in the case of extenuating circumstances at the discretion of the instructor. Please try not to email me at the last minute requesting homework extensions. Your lowest homework grade will be dropped. If a homework assignment is turned in after the due date within 24 hours, it will receive half credit. After that time period, a submission will receive no credit. If you have a medical issue, please provide a doctor's note.

Collaboration and Academic Integrity

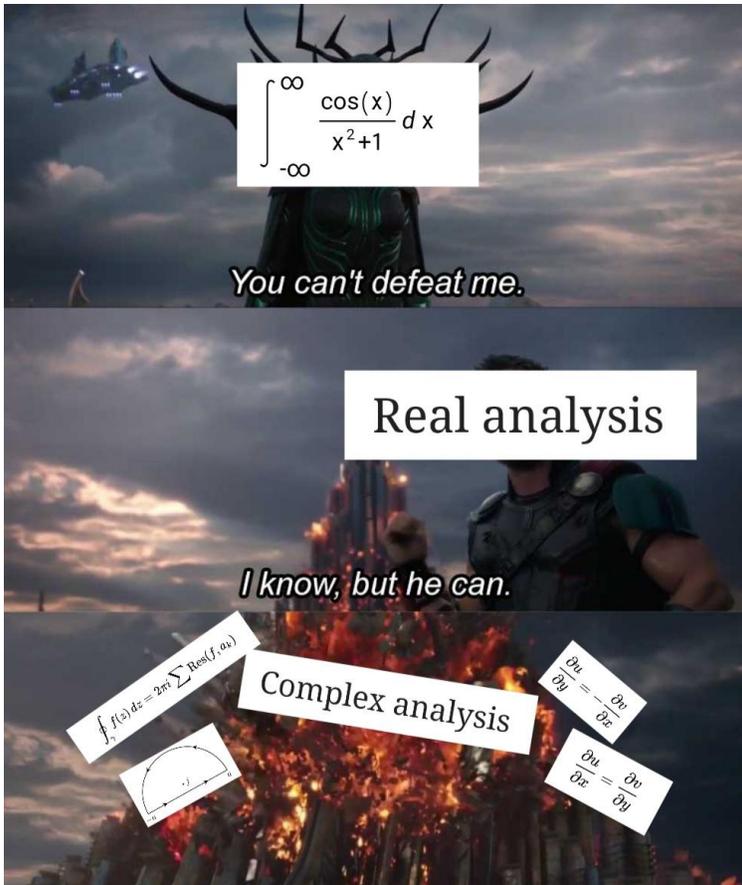
You are highly encouraged to work with others on the homework assignments! Learning mathematics is most effective as a group activity. When you submit your homework on Gradescope, please write the names of any classmates with whom you collaborated on the assignment. **Your submission should represent your own understanding of the material (i.e. do not just copy somebody else's work; you need to be able to explain every step). Please do not use Math Stack Exchange/Overflow to obtain answers to homework exercises.** Please also do not use Chat GPT; AI language models can sometimes offer bizarre and nonsensical answers to proof based mathematics questions anyway. Struggling with a problem can be frustrating, but it helps you learn the material on a very deep level. Also, you are highly encouraged to attend my office hours to discuss homework exercises and obtain hints.

Please be honest and do not cheat! This encompasses your actions with regards to the homework, exams, and final project. You are all expected to follow the Brown University Academic Code <https://college.brown.edu/design-your-education/academic-policies/academic-code>. **Violations of the academic code will automatically result in a grade of zero on the relevant assignment and could lead to further university disciplinary action.** Cheating is a self-defeating action that will never help you learn the underlying mathematics, which is the whole point of this course.

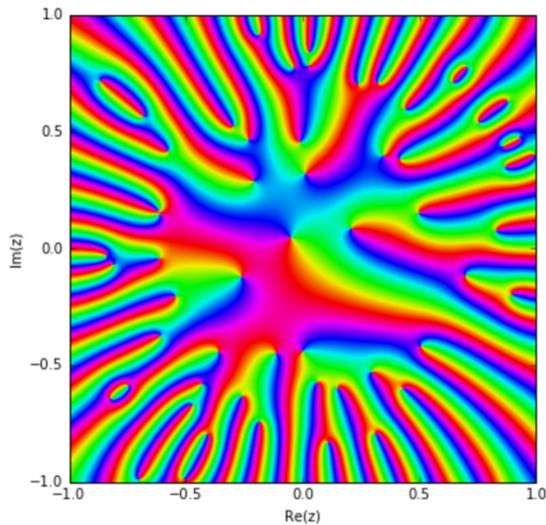
Accommodations for Disabilities and Anti-discrimination Policy

Brown University is committed to full inclusion of all students. Please inform me early in the term if you may require accommodations or modification of any of course procedures. You may speak with me after class, during office hours, or by appointment. If you need accommodations around online learning or in classroom accommodations, please be sure to reach out to Student Accessibility Services (SAS) for their assistance (sas@brown.edu, 401-863-9588). Undergraduates in need of short-term academic advice or support can contact an academic dean in the College by emailing college@brown.edu. Graduate students may contact one of the deans in the Graduate School by emailing graduate_school@brown.edu.

Discrimination of any form (based on sex, race, gender identity, class, ethnicity, religion, etc) will not be tolerated in the classroom. We will strive to create an inclusive and welcoming environment that respects the dignity of all people. Mathematics is a quintessential human endeavor that should be available to everyone, including members of groups that were historically excluded from it because of discrimination. To put it simply, math is for all!



This meme will make sense once we study the residue theorem!



Complex functions can generate beautiful and interesting looking plots, where the color is used to express information about the value (specifically, the “phase”) of a function at particular points.